$$= 2\pi \int_0^1 dz \int_0^t \rho^3 f'(\rho z)\, d\rho .$$

$$= 2\pi \int_0^t d\rho \int_0^1 \rho^3 f'(\rho z)\, dz .$$

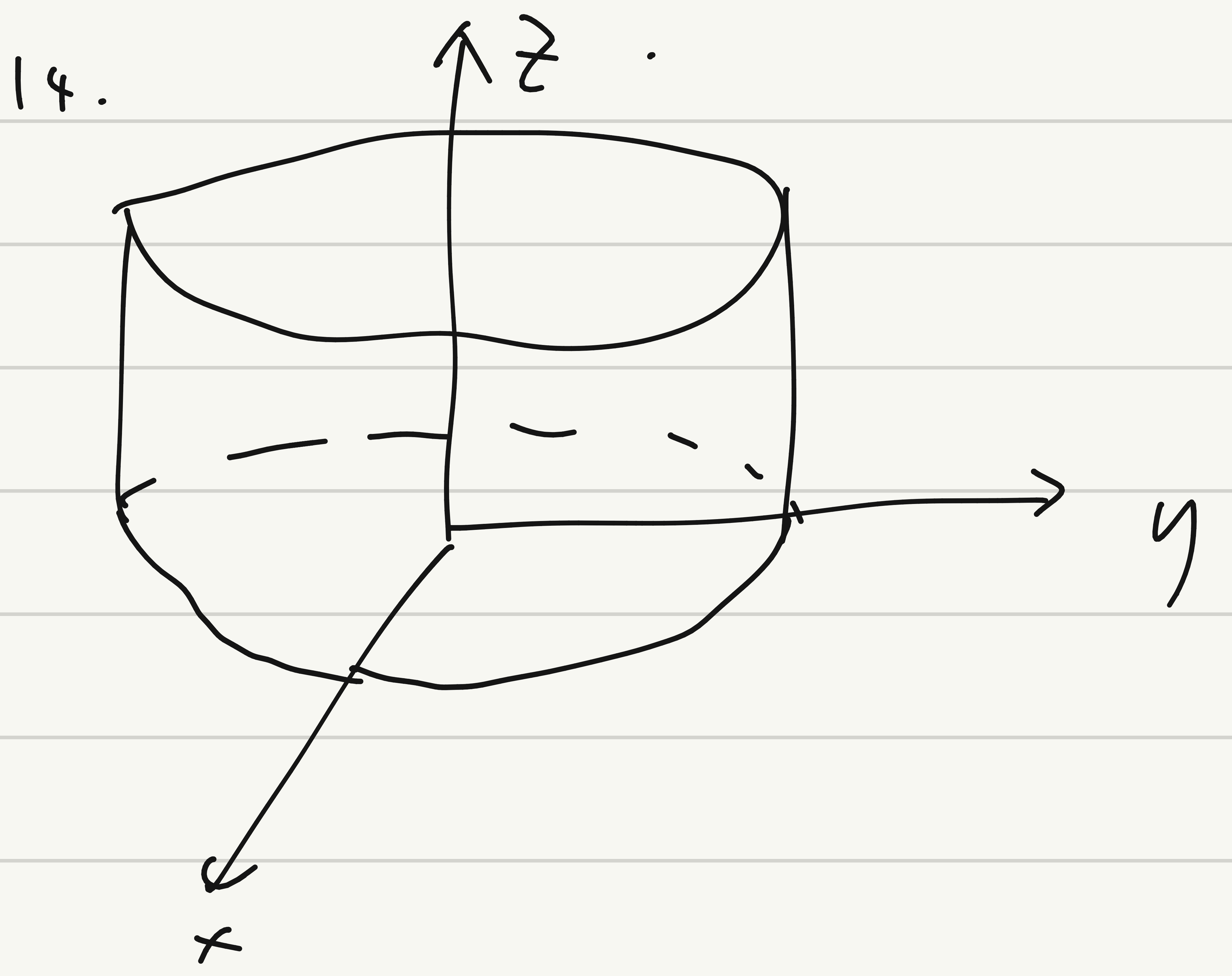
$$= 2\pi \int_0^t d\rho \int_0^\rho \rho^2 f'(\rho z)\, d\rho z .$$

$$= 2\pi \int_0^t \rho^2 [f(\rho) - f(0)]\, d\rho .$$

$$\lim \frac{I_t}{t^4} = \lim \frac{2\pi\, t^2 [f(t) - f(0)]}{4t^3} .$$

$$= \lim \frac{\pi (f(t) - f(0))}{2t} = \frac{\pi}{2} f'(0) .$$

14.



Gass : $\int_{\Omega} 2x+2y+1\,dV$

$= \int_{\Omega} 1\,dV = 4\pi.$

15. 挖球 : $\Sigma' : x^2+y^2+z^2=\varepsilon^2$ 内侧

$P_x = \dfrac{(x^2+y^2+z^2)^{\frac{3}{2}} - x\frac{3}{2}(x^2+y^2+z^2)^{\frac{1}{2}}\cdot 2x}{(x^2+y^2+z^2)^3}$

$= \dfrac{y^2+z^2-2x^2}{(x^2+y^2+z^2)^{\frac{5}{2}}}$

$\int_{\Sigma+\Sigma'} \cdots = 0$

$-\int_{\Sigma'} \dfrac{\underline{x\,dy\,dz} + y\,dz\,dx + z\,dx\,dy}{(x^2+y^2+z^2)^{\frac{3}{2}}}$

$= \frac{1}{\varepsilon^3}\int x\,dy\,dz + \cdots$

$= \frac{1}{\varepsilon^3}\int 3\,dV = \frac{1}{\varepsilon^3}\, 3\cdot\frac{4\pi}{3}\varepsilon^3 = 4\pi.$

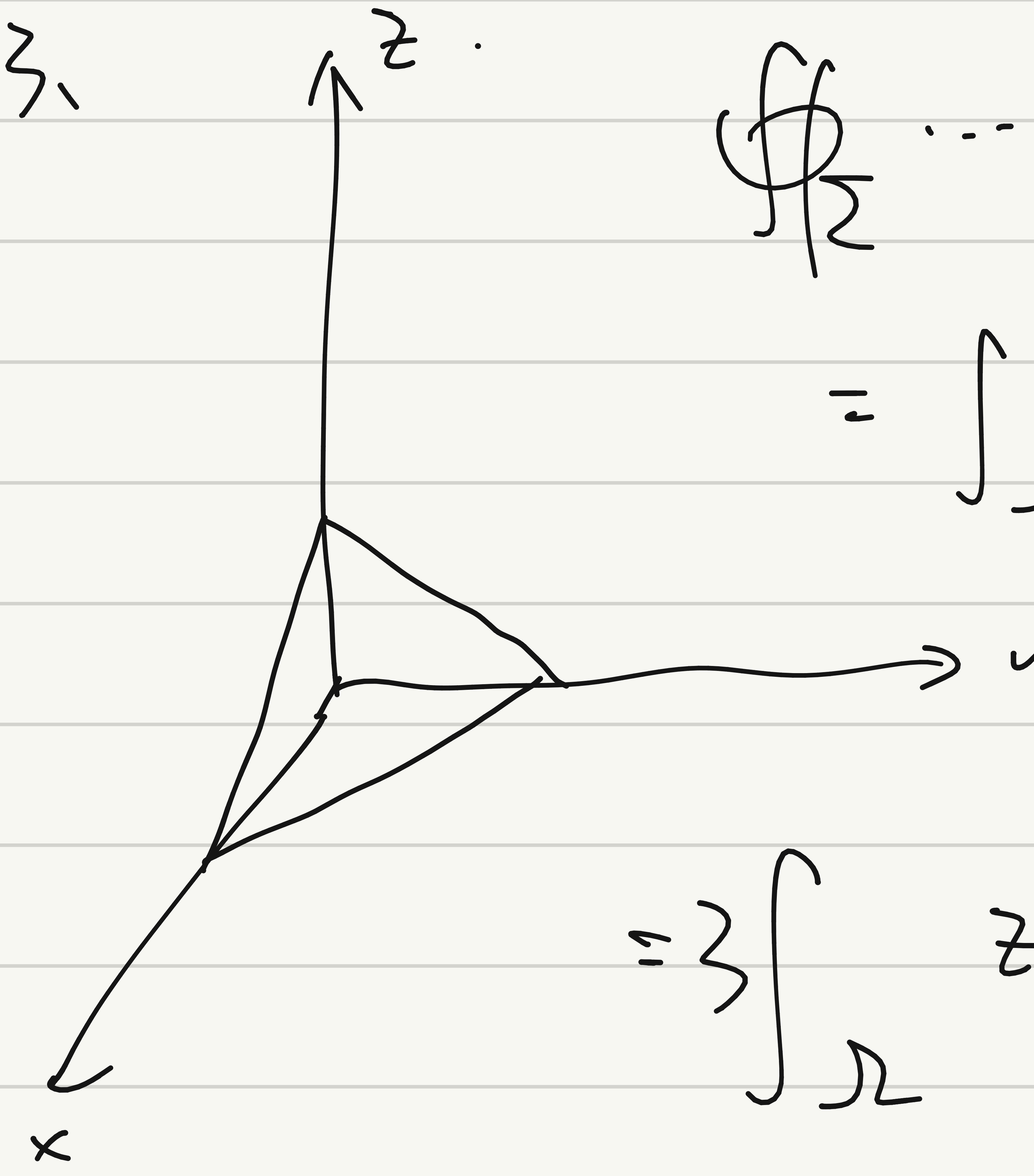
16. $I_t = \int_V f'\cdot 2xz + f'\cdot 2yz + f'\cdot(x^2+y^2)\,dV.$

对称性

$= \int_0^1 dz \int_D (\underline{2xz+2yz} + x^2+y^2)\cdot f'((x^2+y^2)z)\,dV$

$= \int_0^1 dz \int_0^{2\pi} d\theta \int_0^t \rho^3 f'(\rho^2 z)\,d\rho.$

13.



$$\oiint_\Sigma \cdots$$

$$= \int_\Omega x+y+z\,dV.$$

$x+y=1-z$.

$$=3\int_\Omega z\,dV$$

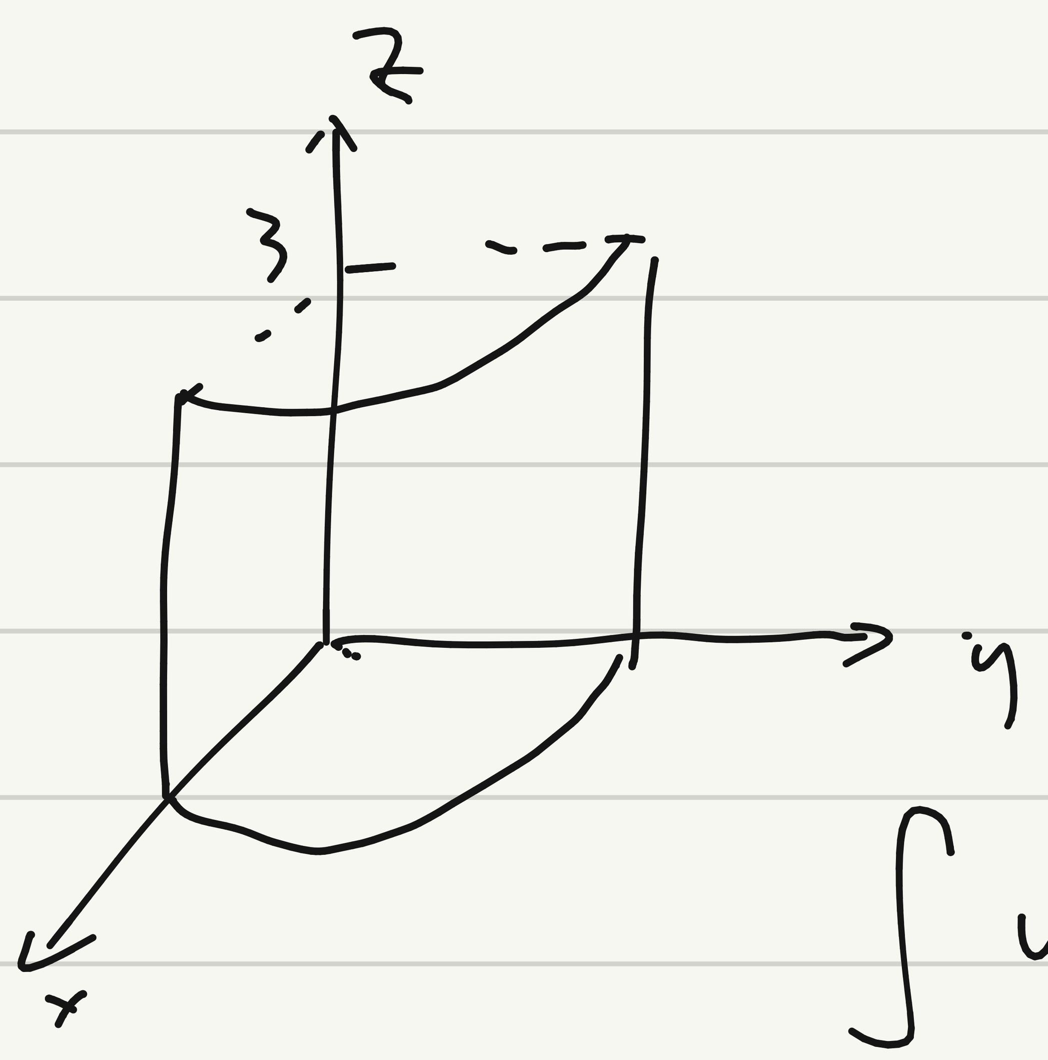
$\frac{(1-z)^2}{2}\cdot z$.

$$=3\int_0^1 dz \underline{\int_D z\,dxdy}.$$

$$=3\int_0^1 \frac{z(1-z)^2}{2}\,dz.$$

$$=\frac{3}{2}\int_0^1 z^2 - z^3\,dz. = \frac{1}{8}$$

13.



$$\int z\,dxdy = 0$$

$$\int x\,dydz = \int_0^3 dz \int_0^1 \sqrt{1-y^2}\,dy$$

$$\int y\,dxdz = \int_0^3 dz \int_0^1 \sqrt{1-x^2}\,dx.$$

$$\frac{3\pi}{2}$$

12、

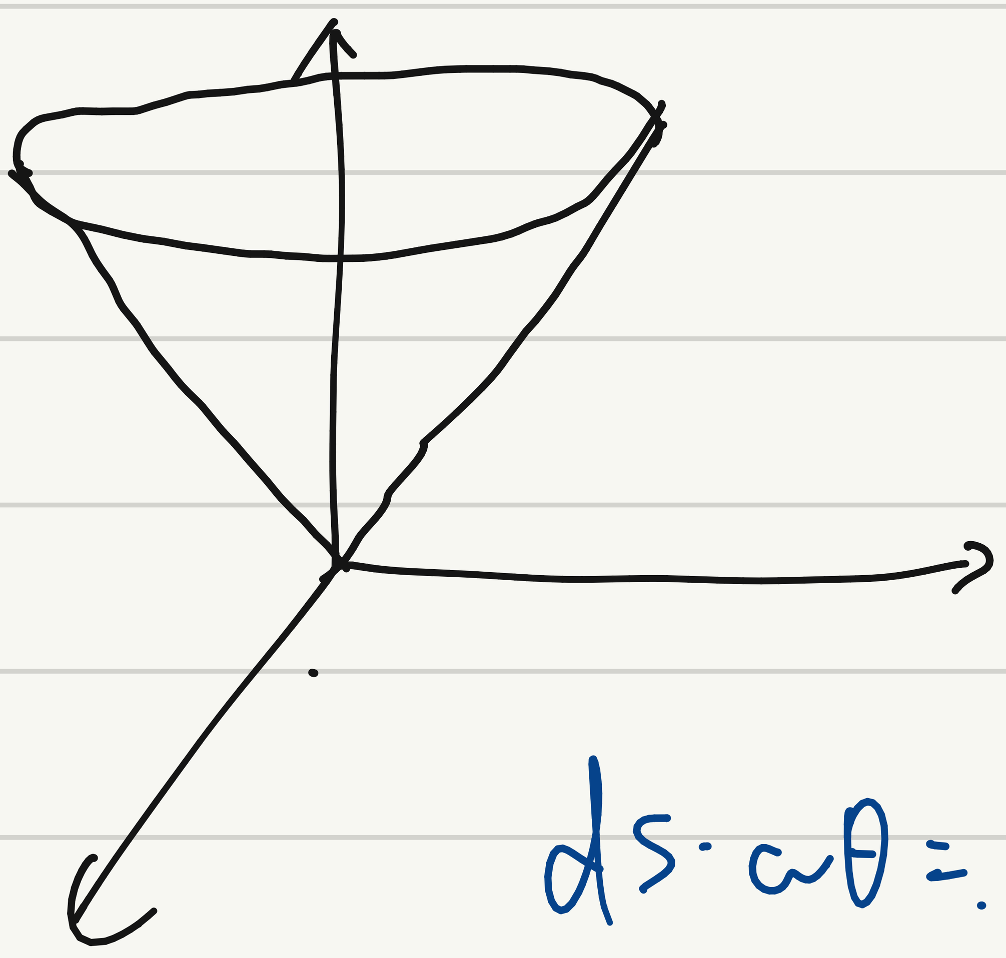
$x^2+y^2=2ax$

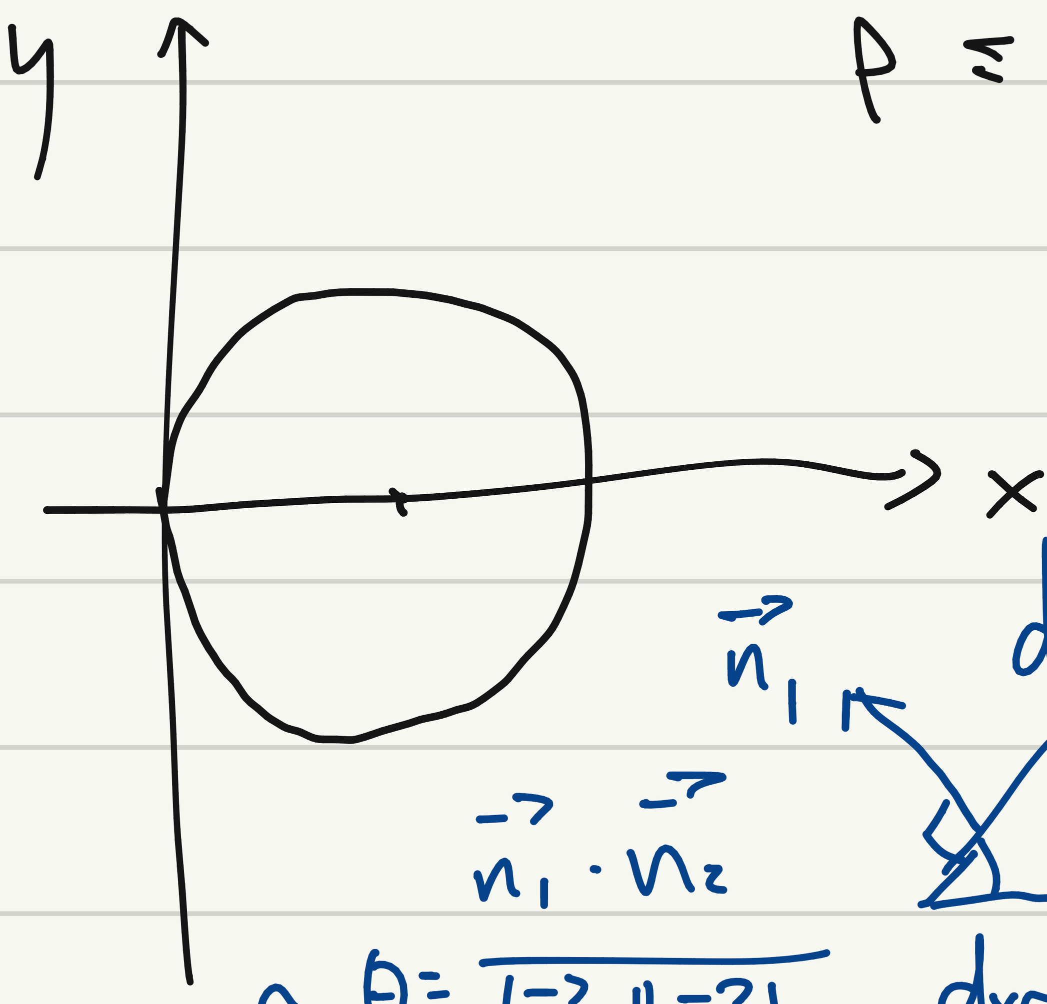
$\rho^2 \le 2a\rho\cos\theta$.

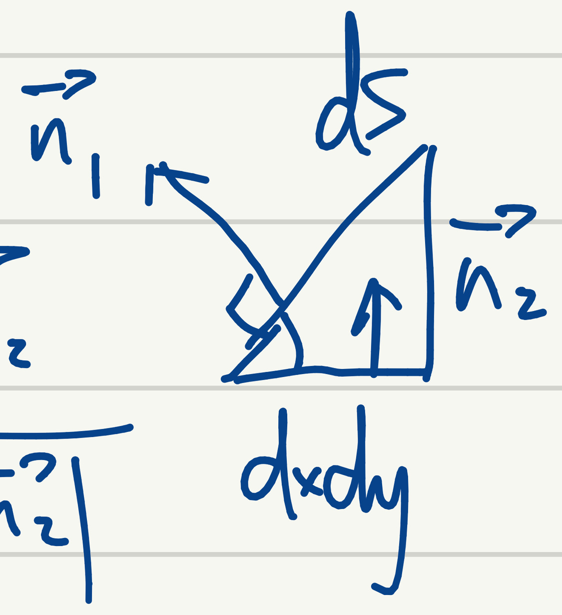
$\rho \le 2a\cos\theta$.

$\vec{n}_1=(z_x, z_y, 1)$

$\vec{n}_2=(0,0,1)$.







$ds\cdot\cos\theta = dxdy$

$\cos\theta=\frac{\vec{n}_1\cdot\vec{n}_2}{|\vec{n}_1||\vec{n}_2|}$

$$\int_{(x-a)^2+y^2\le a^2} (xy+yz+zx)\sqrt{1+(z_x)^2+(z_y)^2}\,dxdy$$

$$=\int [xy+\sqrt{x^2+y^2}(x+y)]\sqrt{2}\,dxdy.$$

$$=\sqrt{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\theta\int_0^{2a\cos\theta} [\rho^2\cos\theta\sin\theta+\rho(\rho(\cos\theta+\sin\theta))]\rho\,d\rho.$$

$$=\sqrt{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\theta\int_0^{2a\cos\theta} \rho^3\cos\theta\sin\theta+\rho^3(\cos\theta+\sin\theta)\,d\rho.$$

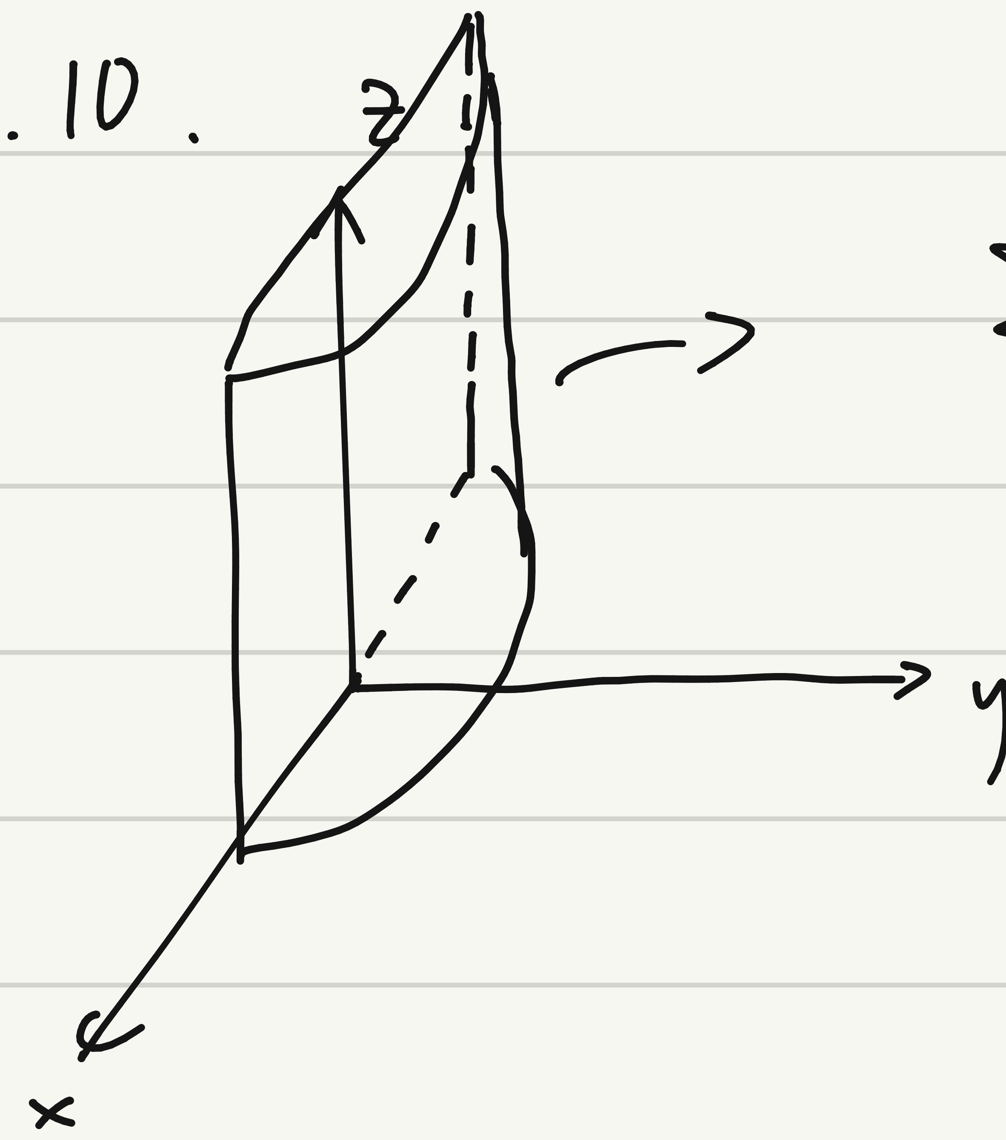
$$=\sqrt{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{(2a\cos\theta)^4}{4}(\cos\theta\sin\theta+\sin\theta+\cos\theta)\,d\theta.$$

$$=\sqrt{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{(2a)^4\cos^5\theta}{4}\,d\theta.$$

$$=\sqrt{2}\cdot 4\cdot 2a^4\int_0^{\frac{\pi}{2}}\cos^5 d\theta$$

$$=8\sqrt{2}a^4\cdot\frac{4}{5}\cdot\frac{2}{3}=\frac{64\sqrt{2}}{15}a^4$$

e.g. 10.



$\Sigma: y = \sqrt{R^2 - x^2}$.

$\Sigma': y = \sqrt{R^2 - x^2} \quad x > 0$.

$$\iint_\Sigma xy\,dydz + xz\,dzdx + \underline{yz\,dxdy}$$

无，向 xy 平面投影为 0.

$$\iint_\Sigma xy\,dydz.$$

$$= \iint_{x=\sqrt{R^2-y^2}} xy\,dydz - \iint_{x=-\sqrt{R^2-y^2}} \underline{xy\,dydz}$$

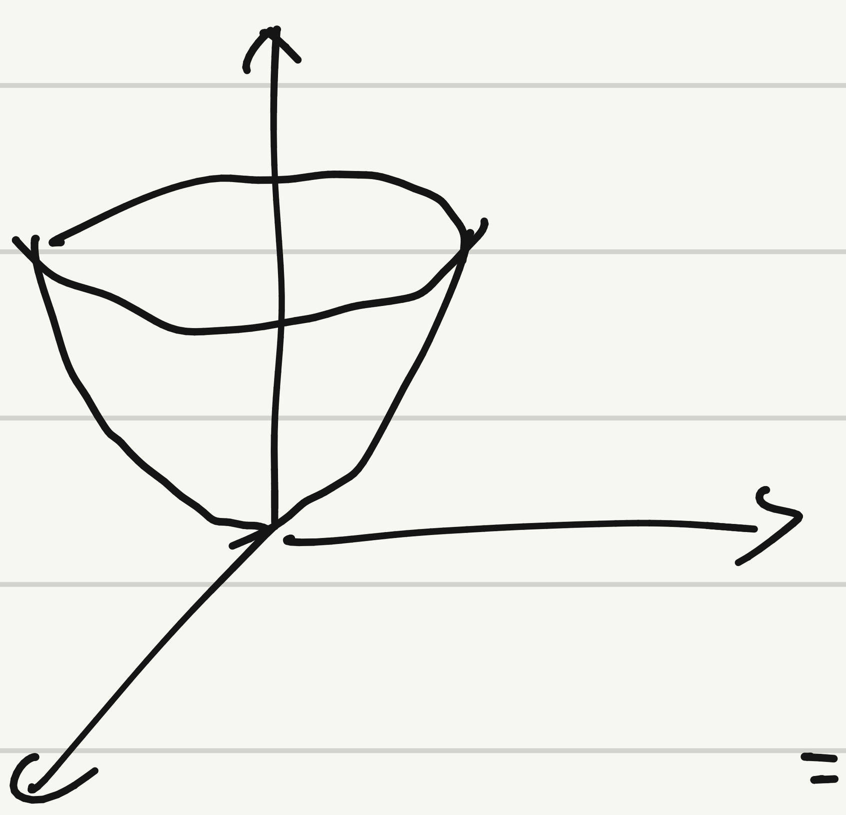
从 x 轴看是下侧

$$= 2\int_0^H dz \int_0^R \sqrt{R^2-y^2}\, y\,dydz.$$

$$= \frac{2}{3}HR^3.$$

$$\int_{-R}^{R} dx \int_0^H xz\,dz = 0$$

e.g. 11



$$\iint_\Sigma |xyz|\,dS = 4\iint_{\Sigma^+} xyz\,dS$$

$$= 4\iint_{x^2+y^2\le 1} xy(x^2+y^2)\sqrt{1+(z_x)^2+(z_y)^2}$$

$$= 4\iint_{x^2+y^2\le 1} xy(x^2+y^2)\sqrt{1+4x^2+4y^2}\,dxdy.$$

$$= 4\int_0^{\frac{\pi}{2}} d\theta \int_0^1 \rho^5 \cos\theta\sin\theta\sqrt{1+4\rho^2}\,d\rho.$$

$$= 4\int_0^{\frac{\pi}{2}} d\theta \int_0^1 \cos\theta\sin\theta\,\rho^5\sqrt{1+4\rho^2}\,d\rho = \frac{1}{4}\left(125\sqrt{5} - \frac{1}{105}\right)$$