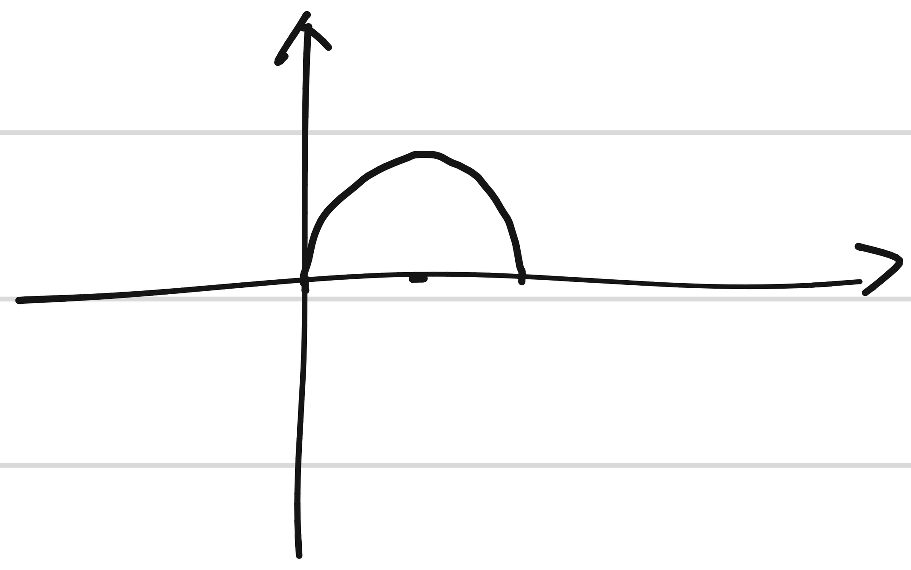
1、 $\int_0^2 x\sqrt{2x-x^2}\,dx$

$= \int_0^2 x\sqrt{x(2-x)}\,dx = \int_0^2 (2-x)\sqrt{x(2-x)}\,dx$

$2I = 2\underbrace{\int_0^2 \sqrt{2x-x^2}}$



$= 2\cdot\frac{1}{2}\pi = \pi.$

$\int_0^2 x\sqrt{2x-x^2} = \int_0^2 x\sqrt{1-(x-1)^2}\,dx.$

$u=x-1$

$= \int_{-1}^{1} (u+1)\sqrt{1-u^2}\,du = \int_{-1}^{1} \underbrace{u\sqrt{1-u^2}}_{\text{奇}} + \underbrace{\sqrt{1-u^2}}_{\text{偶}}\,du.$

2、 $\int_0^\pi x\sqrt{\cos^2x-\cos^4x}\,dx$

$= \int_0^\pi (\pi-x)\sqrt{\cos^2(\pi-x)-\cos^4(\pi-x)}\,dx$

$2I = \pi\int_0^\pi \sqrt{\cos^2x-\cos^4x}\,dx$ ($(\frac{\pi}{2},\pi)$ 上 $\cos x<0$.

$= 2\pi\int_0^{\frac{\pi}{2}} \sqrt{\cos^2x-\cos^4x}\,dx$

$= 2\pi\int_0^{\frac{\pi}{2}} \cos x\sqrt{1-\cos^2x}\,dx$

$= 2\pi\int_0^{\frac{\pi}{2}} \frac{1}{2}\sin 2x\,dx.$

$= \frac{\pi}{2}\cdot(-\cos 2x)\Big|_0^{\frac{\pi}{2}} = \pi.$

3、 $f(-x) = \int_0^{-x} e^{\cos t} dt$.

$\overset{t=-u}{=\!=} \int_0^{x} -e^{\cos u} du = -f(x)$ 奇.

$g(-x) = \int_0^{-\sin x} e^{t^2} dt \overset{t=-u}{=} \int_0^{\sin x} e^{t^2} (-du) = -g(x)$.

周期： $f(x) = f(x+T)$

$\int_0^{x} e^{\cos t} dt = \int_0^{x+T} e^{\cos t} dt$.

$\int_x^{x+T} e^{\cos t} dt = 0$ ×

$g(x+2\pi) = \int_0^{\sin x} \cdots = g(x)$.

4、 $(0, \frac{\pi}{4})$: $\sin x \in (0, \frac{\sqrt{2}}{2})$ $\cos x \in (\frac{\sqrt{2}}{2}, 1)$.

$\cot x = \frac{1}{\tan x} \in (1, +\infty)$

$\ln \cot x > 0$ J最大.

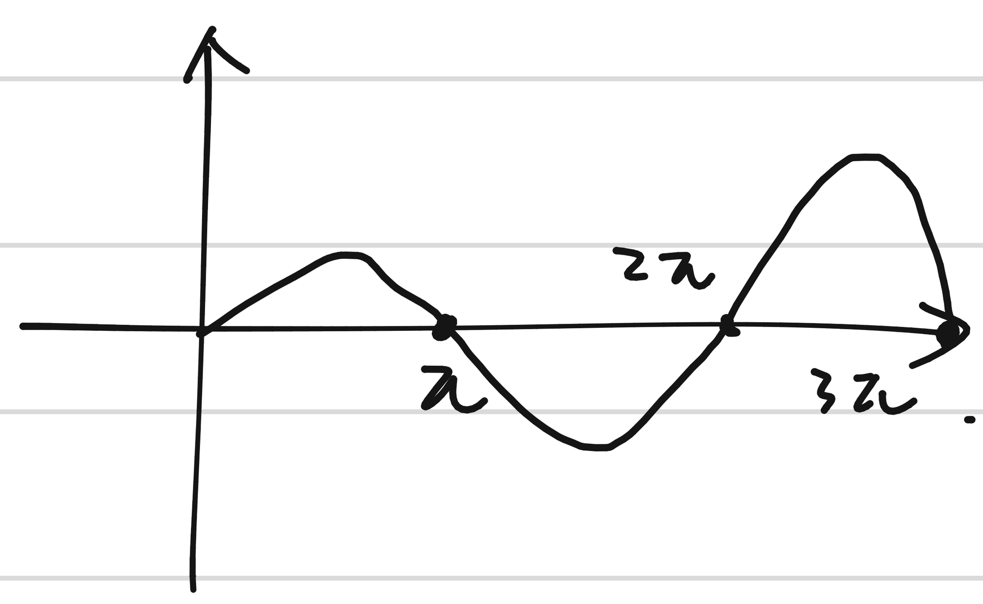
$\sin x < \cos x < 1$ 故 $I < K < J$

5、 $(0, \pi)$: $e^{x^2} \sin x > 0$

$(\pi, 2\pi)$: $e^{x^2} \sin x < 0$

$(2\pi, 3\pi)$: $e^{x^2} \sin x > 0$.

故 $I_1 > I_2$, $I_3 > I_2$.



若 $u \in (\pi, 2\pi)$ $u+\pi \in (2\pi, 3\pi)$.

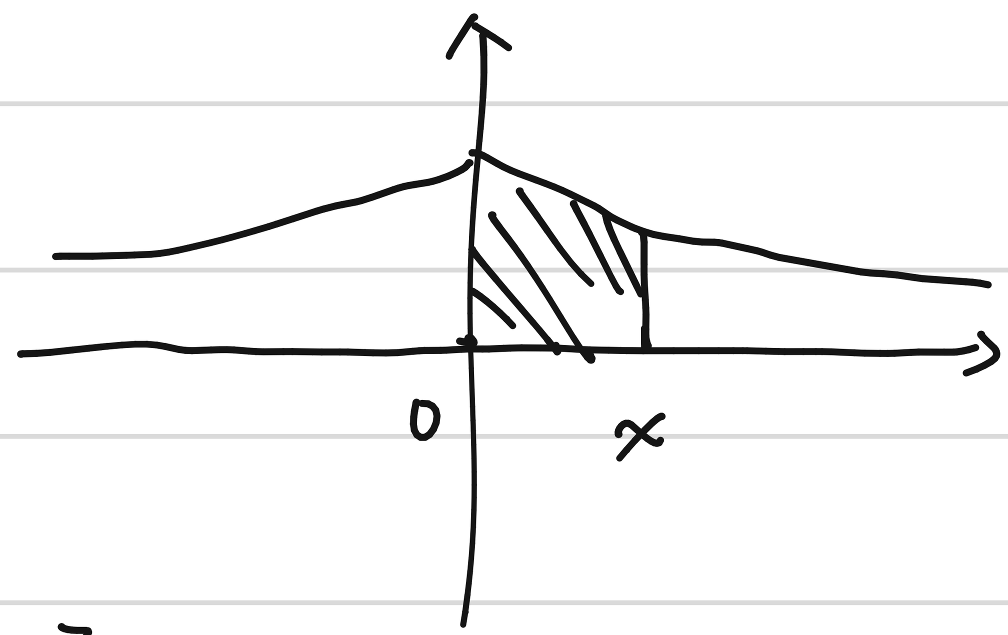
$|e^{u^2} \sin u| < |e^{(u+\pi)^2} \cdot \sin u|$

故 $\left|\int_{2\pi}^{3\pi} \cdots\right| > \left|\int_{\pi}^{2\pi} \cdots\right|$ $I_3 > I_1 > I_2$.

6、$\int_{-\pi}^{\pi} (\cos^2 x + f(x)) \sin^2 x \, dx$.

$= \int_{-\pi}^{\pi} \cos^2 x \sin^2 x \, dx + \int_{-\pi}^{\pi} \underline{f(x) \sin^2 x} \, dx$

$f(x) = -f(-x)$



$I_2 = 0$ $I_1 = 2\int_0^{\pi} \cos^2 x \, (1 - \cos^2 x) dx$

$= 2\int_0^{\pi} \cos^2 x - \cos^4 x \, dx$

$= 4\int_0^{\frac{\pi}{2}} \cos^2 x - \cos^4 x \, dx$.

$\int_0^{\frac{\pi}{2}} \sin^{2n} x \, dx = \frac{2n-1}{2n} \cdot \frac{2n-3}{2n-2} \cdots \frac{1}{2} \cdot \frac{\pi}{2}$.

$\int_0^{\frac{\pi}{2}} \sin^{2n+1} x \, dx = \frac{2n}{2n+1} \cdot \frac{2n-2}{2n-1} \cdots \frac{2}{3} \cdot 1$

7、A: $x^2 (x \cdot e^{-x}) \to 0$. 收敛.

或积出

B: $x^2 (x e^{-x^2}) \to 0$ 收敛.

C: $x^2 \cdot \frac{\operatorname{arcctg} x}{1+x^2} \to \frac{\pi}{2}$ 收敛.

or $d \operatorname{arcctg} x$

D: $\frac{1}{2}\ln(1+x^2)\Big|_0^{+\infty}$. 发散.

8、A. $(0,\frac{1}{2n})$: $f(\frac{1}{2n})$

$(\frac{1}{2n},\frac{2}{2n})$: $f(\frac{3}{2n})$

$(\frac{2}{2n},\frac{3}{2n})$: $f(\frac{5}{2n})$

(包括下标也不正常.)

B: $(0,\frac{1}{n})$: $f(\frac{1}{2n})$

$(\frac{1}{n},\frac{2}{n})$ …… 合理.

C. $(0,\frac{1}{n})$ $\frac{1}{2n}$

$(\frac{1}{n},\frac{2}{n})$ $\frac{2}{2n}$

$(\frac{2}{n},\frac{3}{n})$ $\frac{2}{2n}$. ×

D. 下标不合理.

9. $\sum \frac{1}{n}\frac{e^{\frac{k}{n}}}{1+\frac{1}{nk}} < \sum \frac{1}{n}\frac{e^{\frac{k}{n}}}{1} \to \int_0^1 e^x dx$

$= e-1$

$\sum\left|\frac{1}{n}\frac{e^{\frac{k}{n}}}{1}-\frac{1}{n}\frac{e^{\frac{k}{n}}}{1+\frac{1}{nk}}\right|$

$$= \sum \frac{1}{n} e^{\frac{k}{n}} \left(\frac{\frac{1}{nk}}{1+\frac{1}{nk}} \right)$$

$$= \sum \frac{1}{n} e^{\frac{k}{n}} \cdot \frac{1}{1+nk}$$

$$< \sum \frac{1}{n} e^{\frac{k}{n}} \frac{1}{1+n}$$

$$< \sum \frac{e}{n(n+1)} < \frac{e}{n+1} \to 0 .$$

10、 $\frac{1}{n}\left[\frac{1}{n} \ln \frac{1}{n} + \frac{2}{n} \ln \frac{2}{n} + \cdots + \frac{n-1}{n} \ln \frac{n-1}{n} \right]$

$$\to \int_0^1 x \ln x \, dx$$

$$= \frac{1}{2} x^2 \ln x \Big|_0^1 - \int_0^1 \frac{x}{2} dx = -\frac{1}{4}$$

11、 $\lim \underbrace{\frac{f(x)-3}{x-2}}_{\to -12.} \cdot \underbrace{\frac{1}{x^2+2x+4}}_{\to \frac{1}{12}} = -1 .$

$x-2 \to 0 \quad f(x)-3 \to 0 \quad f(2)=3 .$

$$n \cdot \int_{2-\frac{3}{n}}^{2+\frac{1}{n}} f = n \cdot \frac{4}{n} \cdot f(\xi) \to 4 f(2) = 12 .$$

12. 設 $\int_0^1 x^2 f = C .$

$$f(x) = \sqrt{1-x^2} + \frac{C}{1+x^2}$$

$$\int_0^1 x^2 f = \int_0^1 x^2\sqrt{1-x^2} + \frac{Cx^2}{1+x^2}$$

$$C = \int_0^{\frac{\pi}{2}} \sin^2 x \cdot \cos^2 x \, dx + \underline{\int_0^1 \frac{C(x^2+1-1)}{x^2+1} dx}$$

$\downarrow$

$$\int_0^1 C - \frac{C}{x^2+1} dx$$

$$C - C \cdot \operatorname{arctg}\Big|_0^1$$

$$\frac{\pi}{4} = \int_0^{\frac{\pi}{2}} \cdots$$

13、 $f(0)=0$ $\quad [0,2) : y=(x-1)^2-1$

$$f(7) = f(3) = f(-1) = -f(1) = -(-1) = 1$$

14， $\int_2^3 f(x)dx = \int_0^1 f(x+2)dx$

$$\geq \int_0^1 f(x)+x\,dx$$

$$\underline{1} = \int_0^2 f(x)dx + \int_0^1 x\,dx \geq \frac{1}{2}$$

15， 1°： $F(t) = \int_1^t g(y)dy \qquad g(y) = \int_y^t f(x)dx$

固定 t.

$$= y\cdot g(y)\Big|_1^t - \int_1^t y\cdot$$

$$= t\cdot g(t) - g(1) + \int_1^t y f(y)dy$$

$$= t\cdot \int_t^t f(t)dt - \int_1^t f(x)dx + \int_1^t y f(y)dy.$$

$$F'(t) = -f(t) + t f(t) = (t-1)f(t).$$

2° $h(u) = \int_0^u f(x)dx$

$F(t) = \int_1^t h(t) - h(y)\,dy$

$= h(t)\,(t-1) - \int_1^t h(y)\,dy$

$F'(t) = h'(t)(t-1) + h(t) - h(t).$

$= (t-1)\,f(t).$

16. $f(x) = x - C$

$\cos(\pi - x)$

$= -\cos x$

$f(x)\cos x = x\cos x - C\cdot\cos x$

$C = \int_0^\pi x\cos x = x\sin x\Big|_0^\pi - \int_0^\pi \sin x\,dx$

$= -\int_0^\pi \sin x\,dx = -2.$

$f(x) = x+2$

$\int_0^\pi (x+2)\sin^4 x\,dx$

$I_1 = \int_0^\pi (\pi - x)\sin^4 x\,dx.$

$2I_1 = \pi\cdot\int_0^\pi \sin^4 x\,dx$

$= 2\pi\int_0^{\frac{\pi}{2}} \sin^4 x\,dx.$